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Non-Fourier Dynamic Thermoelasticity with Temperature-Dependent Thermal Properties

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This study numerically investigates the effect of temperature-dependent thermal conductivity, specific heat, and density on coupled, non-Fourier dynamic thermoelastic models involving relaxation times. The thermal response of a semi-infinite slab to both a step change and linear, ramp increase in applied surface temperature is presented. It is observed that a thermal conductivity which decreases with temperature results in reduced temperature gradients, and therefore reduced maximum displacements and stresses. However, decreasing specific heat or density with temperature causes increased temperature gradients, and therefore increased displacements and stresses. In addition to altering the magnitudes of the displacements and stresses, the variable properties also result in phase shifts of the displacement and stress waves. The ramp increase in surface temperature results in displacement and stress waves of lower magnitudes that are spread over a longer time interval compared to the results for the step change in surface temperature.

Nomenclature

B = array defined by Eq. (14e)
C = dimensionless specific heat, $c_p/c_{pr} = (1 + \beta_c\theta)$
C_s = dimensionless speed of stress wave
c = speed of stress wave
c_p = constant pressure specific heat
c_v = constant volume specific heat
E = modulus of elasticity
$\mathbf{E}$ = vector defined by Eq. (14b)
F = vector defined by Eq. (14c)
H = dimensionless enthalpy
K = dimensionless thermal conductivity, $k/k_r = (1 + \beta_k\theta)$
k = thermal conductivity
L = reference slab length, α/c
Q = dimensionless conduction heat flux
R = dimensionless density, $\rho/\rho_r = (1 + \beta_r\theta)$
S = vector defined by Eq. (14d)
T = temperature
T_0 = reference temperature of the stress free state of the solid
t_0, t_1 = relaxation parameters
U = dimensionless displacement
u = displacement
V = dimensionless stress wave velocity
W = negative of dimensionless stress
α = parameter used in generalized MacCormack's method
α_1 = thermal diffusivity
β_c = specific heat coefficient
β_k = thermal conductivity coefficient
β_r = density coefficient
γ = parameter used in generalized MacCormack's method
δ = coupling parameter
η = dimensionless position, $x/2L$
θ = dimensionless temperature, T/T_r
λ = parameter used in generalized MacCormack's method
ν = Courant number, $\Delta\xi/\Delta\eta$
ξ = dimensionless time, $\alpha_1 t/4L^2$
ρ = density
σ = dimensionless stress, $\partial U/\partial\eta$
τ_0, τ_1 = dimensionless relaxation parameters
ϕ = parameter used in generalized MacCormack's method

Subscripts
i = spatial node
r = reference value
s = surface value
x = differentiation with respect to x
0 = initial condition

Superscripts
n = time level
α = parameter used in generalized MacCormack's method
$\dot{}$ = differentiation with respect to time
$\tilde{}$ = predicted time level

Introduction

THE prediction of the dynamic propagation of thermally induced disturbances in solids is of considerable practical importance in engineering and physics. Several pathological anomalies exist for the classical theory of heat conduction involving Fourier's equation, especially for cases involving extremely short transients or very low temperatures near absolute zero. In contrast to the classical thermoelastic models where the temperature disturbances are assumed to propagate at infinite speeds, modified theories involving non-Fourier heat conduction based on the general notion of relaxing the heat flux in the classical theory have been proposed to account

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for the finite speeds of thermal wave propagation and thermally induced stress wave propagation.

A recent symposium on "heat waves"[1] discussed issues of dynamic thermoelasticity with relevance to nonclassical and classical effects in structures and materials. Although earlier investigations starting from that of Danilovskaya[2] were based on the classical theory of thermoelasticity, research has recently been directed towards investigating the effects of heat waves in structures and materials. The concept of the wave nature of thermal energy transport dates to Maxwell[3] and has been derived using different approaches, including that due to Cattaneo.[4] The applicability of the nonclassical theories is well summarized.[5–10] Prevost and Tao[11] employed Green and Lindsay's[10] thermoelasticity model in conjunction with the finite element method to study the effect of thermally induced stress waves in a half-space. Their results indicate that the effects due to heat waves are important, although restricted to short transients. Tamma and Railkar[12] employed specially tailored hybrid transfinite element formulations for a particular, uncoupled class of non-Fourier disturbances, and identified the presence of significant thermally induced stress when the speeds of the thermal and stress waves were equal. For unequal speeds of propagation, the relative magnitudes of the stresses from the classical and non-Fourier models were comparable. More recently, Tamma and Namburu[13] described an effective finite element approach for dynamic thermoelasticity due to non-Fourier effects and studied the behavior of the thermal-stress disturbances for an elastic half-space subjected to a constant and ramp-type heating conditions.

This article investigates the behavior of the propagating thermal and thermally induced stress waves due to the presence of temperature-dependent thermophysical properties. The study employs the dynamic thermoelasticity model of Green and Lindsay.[10] A key feature of this model is that it does not violate the classical heat conduction law if the material has a center of symmetry at each point, and the theory is based on an entropy production inequality proposed by Green and Laws.[14] Since closed-form solutions are impractical for the dynamic thermoelasticity problems under consideration, this article focuses on numerical solution techniques. Numerical solution techniques for solving coupled non-Fourier dynamic thermoelasticity problems have been presented in Refs. 11–13. However, previous investigations considered only constant thermophysical properties. Unlike past efforts, the present study accounts for temperature-dependent thermophysical properties, and consequently, investigates the nature and behavior of the thermal and thermally induced stress-wave disturbances. A generalized explicit MacCormack's predictor-corrector scheme is used to investigate the effects of temperature-dependent thermal properties on an elastic half-space influenced by a step change and a ramp increase in the applied surface temperature.

Mathematical Formulation

The problem under consideration concerns an elastic half-space with a constant initial dimensionless temperature of zero. A step or ramp increase to a constant temperature is applied at the boundary, $\eta = 0$, for times greater than zero. The thermal conductivity, specific heat, and density are assumed to be linear functions of temperature, while the thermal relaxation times are assumed to be constant. Body radiation is neglected.

Governing Equations

The dynamic thermoelasticity response of structures requires the simultaneous solution of the coupled non-Fourier heat conduction equations and the equation of motion. The dynamic thermoelastic model due to Green and Lindsay[10] is considered here. In the absence of body forces and internal heat sources, the coupled form of the thermoelastic model in dimensional form is represented as

$$\rho c_p t_0 \ddot{\theta} + \rho c_v \dot{\theta} + T_0 E \alpha_1 \dot{u}_{,x} - k\theta_{,xx} = 0 \tag{1a}$$

$$\rho \ddot{u} + E\alpha_1 \theta_{,x} + t_1 E \alpha_1 \dot{\theta}_{,x} - E u_{,xx} = 0 \tag{1b}$$

The equations are nondimensionalized following Prevost and Tao.[11] The finite difference solution technique chosen here uses a system of first-order equations. Thus, the energy and non-Fourier heat flux equations are not combined, as was done in Eq. (1a). In addition, the equation of motion [Eq. (1b)] is cast as a system of two first-order equations.

Non-Fourier heat conduction formulations consist of the energy equation and the non-Fourier heat flux equation. Introducing the enthalpy formulation, the dimensionless energy equation takes the form

$$\frac{\partial H}{\partial \xi} = -2\frac{\partial Q}{\partial \eta} - \delta \frac{\partial^2 U}{\partial \eta \partial \xi} \tag{2a}$$

where H is defined as

$$H = \int_0^\theta R(\theta')C(\theta')\,\mathrm{d}\theta' \tag{2b}$$

In addition, δ is a dimensionless parameter coupling the thermal and stress solutions. The specific heat and density are taken to be linear functions of temperature that are expressed as

$$C = (1 + \beta_c\theta) \tag{3a}$$

$$R = (1 + \beta_r\theta) \tag{3b}$$

where β_c and β_r are dimensionless coefficients used to describe temperature-dependent properties. In numerically solving Eq. (2a), it is convenient to replace the time derivative appearing on the right side of the equation. To accomplish this, a dimensionless velocity is used and is defined as

$$V = \frac{\partial U}{\partial \xi} \tag{4}$$

Hence, the energy equation [Eq. (2a)] becomes

$$\frac{\partial H}{\partial \xi} + \frac{\partial (Q + \delta V)}{\partial \eta} = 0 \tag{5}$$

The non-Fourier heat flux equation is

$$\tau_0 \frac{\partial Q}{\partial \xi} + Q + (1 + \beta_k\theta)\frac{\partial \theta}{\partial \eta} = 0 \tag{6}$$

where τ_0 is the dimensionless thermal relaxation time, and β_k is the dimensionless coefficient used in the temperature-dependent thermal conductivity. The thermal conductivity is also assumed to be a linear function of temperature that is expressed as

$$K = (1 + \beta_k\theta) \tag{7}$$

When the relaxation time $\tau_0 = 0$, the non-Fourier heat flux equation [Eq. (6)] reduces to the classical Fourier heat flux equation given as

$$Q = -(1 + \beta_k\theta)\frac{\partial \theta}{\partial \eta} \tag{8}$$

In addition to the energy and flux equations, one must also include the equation of motion, which in dimensionless form is given by

$$\frac{\partial^2 U}{\partial \xi^2} + \frac{\tau_1 C_s^2}{2R}\frac{\partial^2 \theta}{\partial \eta \partial \xi} - \frac{C_s^2}{R}\frac{\partial^2 U}{\partial \eta^2} + \frac{C_s^2}{R}\frac{\partial \theta}{\partial \eta} = 0 \tag{9}$$

where τ_1 is a relaxation time and C_s is the dimensionless stress wave velocity. In the numerical solution of the equation of motion by MacCormack's method, it is convenient to cast the equation into two first-order partial differential equations rather than one second-order equation. Using Eq. (4), as well as defining the negative of the dimensionless stress as

$$W = -\frac{\partial U}{\partial \eta} \tag{10}$$

Equation (9) can be cast into two, first-order partial differential equations as

$$\frac{\partial V}{\partial \xi} = -\frac{C_s^2}{R}\frac{\partial}{\partial \eta}\left(W + \theta + \frac{\tau_1}{2}\frac{\partial \theta}{\partial \xi}\right) \tag{11a}$$

$$\frac{\partial W}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0 \tag{11b}$$

The time derivative can be eliminated from the right side of Eq. (11a) by expressing the energy equation in terms of the temperature and substituting it into Eq. (11a) to yield

$$\frac{C_s^2}{R}\frac{\partial}{\partial \eta}\left\{W + \theta - \frac{\tau_1}{RC}\left[\frac{\partial}{\partial \eta}(Q + \delta V)\right]\right\} + \frac{\partial V}{\partial \xi} = 0 \tag{12}$$

Equation (11b) is now used in conjunction with Eq. (12).

Boundary and Initial Conditions

The dimensionless boundary and initial conditions for the dynamic thermoelastic problem involving relaxation times defined above, result from both physical and numerical aspects of the problem. The physical boundary and initial conditions for a step increase in surface temperature are given for a half-space by

$$\theta = \theta_s \qquad \eta = 0, \quad \xi > 0 \tag{13a}$$

$$\sigma = 0 \quad (\text{or } W = 0) \qquad \eta = 0, \quad \xi > 0 \tag{13b}$$

$$\theta = 0 \qquad 0 \le \eta, \quad \xi = 0 \tag{13c}$$

$$\sigma = 0 \quad (\text{or } W = 0) \qquad 0 \le \eta, \quad \xi = 0 \tag{13d}$$

For the case of a ramp increase in surface temperature, Eq. (13a) is replaced with

$$\theta = \theta_s(\xi/\Delta\xi) \qquad \eta = 0, \quad \xi < \Delta\xi$$

$$\theta = \theta_s \qquad \eta = 0, \quad \xi \ge \Delta\xi \tag{13e}$$

To form a complete mathematical set of equations for the non-Fourier dynamic thermoelasticity problem [Eqs. (5), (6), (11b), and (12)], boundary and initial conditions for Q and V are also necessary. The boundary conditions used in the numerical simulations are obtained by requiring that energy conservation be satisfied at the boundary. Thus, the additional conditions are given by

$$\frac{\partial H}{\partial \xi} = -\frac{\partial(Q + \delta V)}{\partial \eta} \qquad \eta = 0, \quad \xi > 0 \tag{13f}$$

$$\frac{\partial W}{\partial \xi} = -\frac{\partial V}{\partial \eta} \qquad \eta = 0, \quad \xi > 0 \tag{13g}$$

$$Q = 0 \qquad 0 \le \eta, \quad \xi = 0 \tag{13h}$$

$$V = 0 \qquad 0 \le \eta, \quad \xi = 0 \tag{13i}$$

Equations (5), (6), (11b), and (12) together with the boundary and initial conditions [Eqs. (13a–13i)], constitute the complete mathematical formulation for the non-Fourier dynamic thermoelasticity problem.

Numerical Formulation

In numerically simulating the effects of the propagating thermal-stress wave front in the non-Fourier dynamic thermoelasticity problem, it is convenient to solve the energy, flux, and equilibrium equations as a system of first-order partial differential equations, rather than two combined second-order partial differential equations. MacCormack's predictor-corrector scheme, having been validated for hyperbolic heat conduction problems,[15] has been used successfully to model the propagating thermal disturbances for thermal problems with temperature-dependent thermophysical properties (i.e., no stress waves included)[16,17] and is thus chosen for the present analysis. To apply MacCormack's method to the non-Fourier dynamic thermoelasticity problem, Eqs. (5), (6), (11b), and (12) are written in vector form as

$$\frac{\partial E}{\partial \xi} + [B]\frac{\partial F}{\partial \eta} + S = 0 \tag{14a}$$

where

$$E = \begin{bmatrix} H \\ Q \\ V \\ W \end{bmatrix} \tag{14b}$$

$$F = \begin{bmatrix} Q + \delta V \\ \theta \\ \left\{W + \theta - \frac{\tau_1}{RC}\left[\frac{\partial}{\partial \eta}(Q + \delta V)\right]\right\} \\ V \end{bmatrix} \tag{14c}$$

$$S = \begin{bmatrix} 0 \\ Q/\tau_0 \\ 0 \\ 0 \end{bmatrix} \tag{14d}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1 + \beta_k \theta}{\tau_0} & 0 & 0 \\ 0 & 0 & \frac{C_s^2}{R} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14e}$$

To obtain stable solutions for the displacements and stresses, a generalization of MacCormack's predictor-corrector method is used, as presented by Klopfer and McRae.[18] Whereas, in MacCormack's method, the predicted values are obtained at time level $n + 1$, the generalization of MacCormack's method obtains the predicted values at time $n + \alpha$, where $0 < \alpha$. Using the generalized MacCormack's method, the finite difference form of Eq. (14a) is given by Eqs. (15a) and (15b),[18] where the subscript i denotes the grid points in the space domain, superscript n denotes the time level, the superscript tilde refers to the predicted value at time level $n + 1$, and γ, ϕ, and λ are parameters determining whether forward or backward differencing is to be used. Equations (15a) and (15b) are quite general, and can be used for forward or backward differencing (or some combination of the two) in both the predictor and the corrector. In the present analysis, conventional forward differencing is used in the predictor and backward differencing is used in the corrector. This differencing results from values of $\gamma = 0$, $\phi = 1$, and $\lambda = 1$.

Predictor

$$\tilde{\boldsymbol{E}}_i^{n+\alpha} = \boldsymbol{E}_i^n - \alpha\nu(B_i^n)[(1 - \gamma)(\boldsymbol{F}_{i+1}^n - \boldsymbol{F}_i^n) + \gamma(\boldsymbol{F}_i^n - \boldsymbol{F}_{i-1}^n)] - \alpha\Delta\xi \boldsymbol{S}_i^n \tag{15a}$$

Corrector

$$\boldsymbol{E}_i^{n+1} = \boldsymbol{E}_i^n - \frac{\nu}{2\alpha}(2\alpha - 1)(B_i^n)\{(1 - \phi)[\boldsymbol{F}_{i+1}^n - \boldsymbol{F}_i^n] + \phi[\boldsymbol{F}_i^n - \boldsymbol{F}_{i-1}^n]\} - \frac{1}{2\alpha}\{\nu(\tilde{B}_i^{n+\alpha})[(1 - \lambda)(\tilde{\boldsymbol{F}}_{i+1}^{n+\alpha} - \tilde{\boldsymbol{F}}_i^{n+\alpha}) + \lambda(\tilde{\boldsymbol{F}}_i^{n+\alpha} - \tilde{\boldsymbol{F}}_{i-1}^{n+\alpha})] + 2\alpha\Delta\xi\tilde{\boldsymbol{S}}_i^{n+\alpha}\} \tag{15b}$$

To evaluate the accuracy of the above scheme, an analysis of the modified equation is necessary. The modified equation[18,19] is the equation that is actually solved in the numerical solution of the partial differential equation by the differencing scheme given by Eqs. (15a) and (15b). Assume a one-dimensional wave equation with a source term, given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} + S = 0 \tag{16a}$$

Following standard strategies for obtaining the modified equation,[19] the modified equation for Eq. (16a) when using the difference equations given in Eqs. (15a) and (15b) is given, to second order, by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} + S = \frac{\Delta x[-3\lambda + (3 - 6\alpha)\phi + 3\alpha]}{6\alpha}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2(3\alpha\nu\gamma - \alpha\nu^2 + \Psi\nu + \alpha)}{6\alpha}\frac{\partial^3 u}{\partial x^3} + O[(\Delta x)^3, (\Delta x)^2\Delta t, \Delta x(\Delta t)^2, (\Delta t)^3] \tag{16b}$$

where $\Psi = [(3\alpha - 3)\lambda + (3 - 6\alpha)\phi]$.

The traditional MacCormack's method is a second-order accurate scheme in both time and space.[19] To obtain MacCormack's method from the above generalized formulation [Eqs. (15a) and (15b)], in addition to setting $\alpha = 1$, one must require

$$\lambda = \alpha + (1 - 2\alpha)\phi \tag{17}$$

This would make the scheme second-order accurate by eliminating the first term on the right side of Eq. (16b). The terms on the right side of Eq. (16) are referred to as the error terms. The even derivative error term (i.e., the second derivative with respect to x) results in a dissipative error, while the odd derivative error term results in a dispersive error. The dispersive error terms add oscillations to the solution, which tend to decrease the stability of the scheme. In the present study, a stable solution was obtained by eliminating the dispersive error terms (odd derivative, second-order) but leaving the dissipative error terms (even derivative, first-order) in the solution. When forward differencing is used in the predictor ($\gamma = 0$) and backward differencing is used in the corrector ($\phi = \lambda = 1$), the dispersive error term can be eliminated by defining α as

$$\alpha = \frac{3\nu}{3\nu + 1 - \nu^2} \tag{18}$$

Thus, the above generalized MacCormack's scheme, when Eq. (18) is met, allows dissipation but removes dispersion from the solution. Because of the hyperbolic wave nature of both the thermal and thermally induced stress-wave, numerical solution approaches most often lead to oscillatory solution behavior, especially in the vicinity of sharp discontinuities. In the present study, the generalized MacCormack's method is adapted for the coupled nonlinear thermal stress problem. The removal of the dispersive term greatly stabilized the dynamic stress and displacement responses, including the transient thermal behavior.

After each predictor or corrector calculation, the temperatures are obtained from the enthalpy by use of Eq. (2b), and the displacements are obtained from the velocity by use of Eq. (4). The boundary conditions, Eqs. (13a) and (13b) or Eqs. (13b) and (13e), and the initial conditions, Eqs. (13c) and (13d) and Eqs. (13h) and (13i), are applied explicitly in the numerical scheme. The remaining boundary conditions, Eqs. (13f) and (13g), are integrated using MacCormack's predictor and corrector steps in a manner similar to that used for the equations within the medium. However, forward differencing is used at $\eta = 0$ for both the predictor and the corrector.

Problem Description

In the analysis which follows, investigations of the one-dimensional non-Fourier dynamic thermoelasticity equations are carried out for a semi-infinite slab with an applied surface temperature at the boundary $\eta = 0$. Two different cases are studied. The first involves a step change in the surface temperature at $\xi = 0$ from $\theta_s = 0$ to $\theta_s = 1$; and the second involves a linear ramp increase in the surface temperature (again from $\theta_s = 0$ to $\theta_s = 1$) from time $\xi = 0$ to time $\xi = 0.25$. The initial temperature is taken to be $\theta_0 = 0$. The surface at $\eta = 0$ is assumed to be stress free, and thus, $\sigma = 0$. The thermal conductivity is taken to be a function of temperature as $K = (1 + \beta_k\theta)$, where $\beta_k = 0, \pm 0.25$, the specific heat is a function of temperature as $C = (1 + \beta_c\theta)$, where $\beta_c = 0, \pm 0.25$, and the density is a function of temperature as $R = (1 + \beta_r\theta)$, where $\beta_r = 0, \pm 0.25$. In addition, C_s is taken to be unity.

The representative numerical data for the models is based on those employed by Prevost and Tao[11] and Tamma and Namburu,[13] where the medium is assumed to be stainless steel. Although constant thermophysical properties were used in past investigations, the values shown in this article are representative of nonlinear situations (temperature-dependent thermophysical properties). The parameters are nondimensionalized in accordance with Prevost and Tao.[11] As a result, the relaxation parameters τ_0 and τ_1 take on the value 0.971693. The coupling parameter between the thermal and stress solution, δ, is taken to be $\delta = 1.5548 \times 10^{-5}$. For all cases, ν is taken to be $\nu = 0.72$, and $\alpha = 0.81769$ [defined by Eq. (18)]. The results obtained have been validated by means of numerical experiments which accurately simulate the thermal counterpart for which exact solutions are available,[20] as well as the stress and displacement solutions given by Tamma and Namburu.[13] The finite difference solutions have also been validated for the case of nonlinear thermal properties by comparison to Ref. 16 for the thermal response. Numerical studies show that a refined mesh of 251 nodes in the region $0 \leq \eta \leq 2$ gives acceptable results and agrees with past studies. Comparisons have also been made with finite difference and finite element codes to validate the results.

For constant properties ($\beta_k = \beta_c = \beta_r = 0$), the results for both test cases agree precisely in regard to the temperature distribution, and qualitatively with the stress and displacement distributions presented by Tamma and Namburu.[13] Minor differences which may arise in the stress and displacement are due to the different numerical techniques employed. As will be seen when studying the effects of the temperature-dependent properties, the stress and displacement are highly dependent on the thermal behavior.

Numerical Results and Discussion

A generalization of MacCormack's explicit predictor-corrector scheme is used to numerically solve the non-Fourier dynamic thermoelasticity problems. Two different cases are studied: the first involves a step change in the surface temperature at $\xi = 0$ from $\theta_s = 0$ to $\theta_s = 1$, and the second involves a linear ramp increase in the surface temperature to $\theta_s = 1$ over the time interval $0 \le \xi \le 0.25$.

Step Increase in Surface Temperature

The first case considered involves a step increase in the surface temperature from $\theta_s = 0$ to $\theta_s = 1$ at time $\xi = 0$. The effect of temperature-dependent thermal conductivity on the temperature, displacement, and stress as a function of time at a nondimensionalized location $\eta = 1$ and at a dimensionless time $\xi = 1$ is shown in Fig. 1. As β_k decreases, and thus as the thermal conductivity decreases with temperature, the thermal front is both delayed in time (phase shift) and decreased in magnitude and gradient (Fig. 1a). (Since the temperature is shown as a function of time, the reference to gradients refers to time. However, since the wave fronts are propagating through the medium, an increased thermal gradient with respect to time is associated with an increased thermal gradient with respect to space.) The phase shift is also noticed in the displacement and stress distributions, but to a lesser extent. However, the maximum value of the displacement (Fig. 1b) and stress (Fig. 1c) are both significantly decreased due to the reduced temperature gradient as a result of the thermal conductivity decreasing with temperature. It is therefore obvious that both the stress and displacement are strongly dependent on the gradient of the thermal front.

The effect of temperature-dependent specific heat on the temperature, displacement, and stress at $\eta = 1$ for a constant step change in the surface temperature applied at time $\xi = 0$ is shown in Fig. 2. The effect of the temperature-dependent specific heat on the temperature distribution is opposite that of temperature-dependent thermal conductivity. As the specific heat decreases with temperature, the thermal gradient increases, resulting in an increase in both the displacement and stress. This opposite effect can be qualitatively confirmed by an analysis of the energy and flux equations. In the energy equation, the specific heat C appears (using the temperature formulation) in the denominator of the coefficient multiplying the spacial derivative term. This coefficient is directly related to the thermal propagation speed. In the non-Fourier heat flux equation, however, the thermal conductivity K appears in the numerator of the coefficient multiplying the spacial derivative term. Thus, the opposite effect on the wave speed is observed by analysis of the governing equations. As β_k increases, and as the thermal conductivity increases with temperature, the thermal front is both delayed in time and decreased in magnitude and gradient, as shown in Fig. 2a. This phase shift is also noticed in the displacement and stress distributions (Figs. 2b and 2c). Though the peak displacement varies little as a function of specific heat, the maximum stress is significantly decreased as a result of specific heat increasing with temperature, i.e., reduced thermal gradients.

The effect of temperature-dependent density on the temperature, displacement, and stress at $\eta = 1$ for a constant step change in the surface temperature applied at $\xi = 0$ is shown in Fig. 3. The temperature distribution is very similar to the variable specific heat temperature distribution. For the thermal problem alone, the effect of variable density would be the same as the effect of variable specific heat since the two parameters appear together as a product. However, this is not the case for coupled thermoelasticity problems since the density appears without the specific heat in the equilibrium equation. There is a greater phase shift in both the displacement and stress due to the variable density than the specific heat. In addition to the greater phase shift, a greater difference is noticed in the magnitude of the displacement with a variable density than with a variable thermal conductivity or specific heat. The greater effect of the density (compared to the thermal conductivity and specific heat) on the stress and displacement is due to the stronger influence of the density in the equation of motion, Eq. (12). Both the specific heat and density have little effect on the value of the temperature after the thermal front has passed. This effect is in contrast

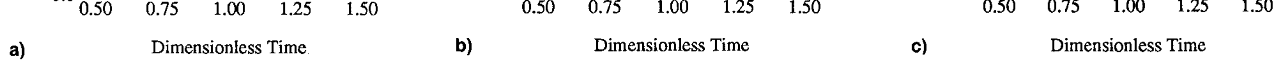

Fig. 1 Effect of temperature-dependent thermal conductivity on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a step increase in surface temperature to $\theta_s = 1$.

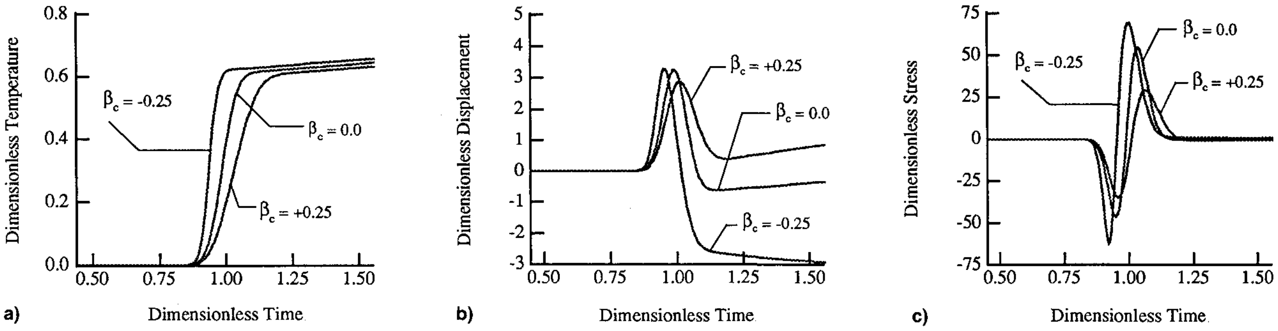

Fig. 2 Effect of temperature-dependent specific heat on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a step increase in surface temperature to $\theta_s = 1$.

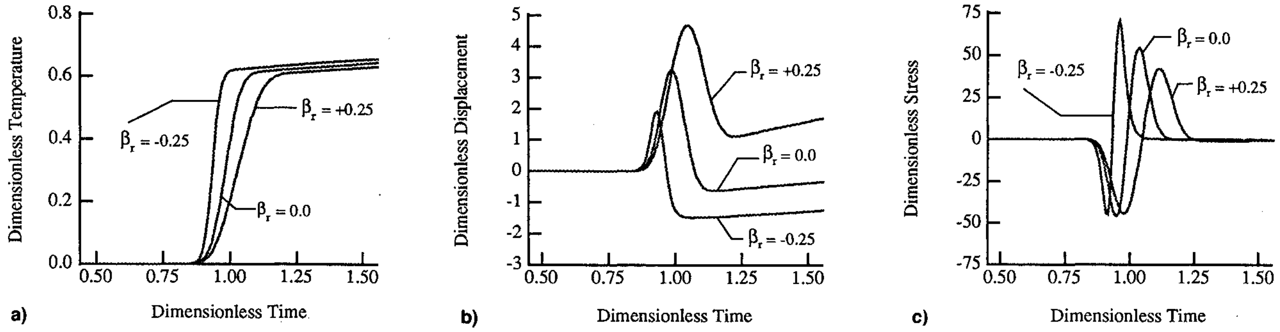

Fig. 3 Effect of temperature-dependent density on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a step increase in surface temperature to $\theta_s = 1$.

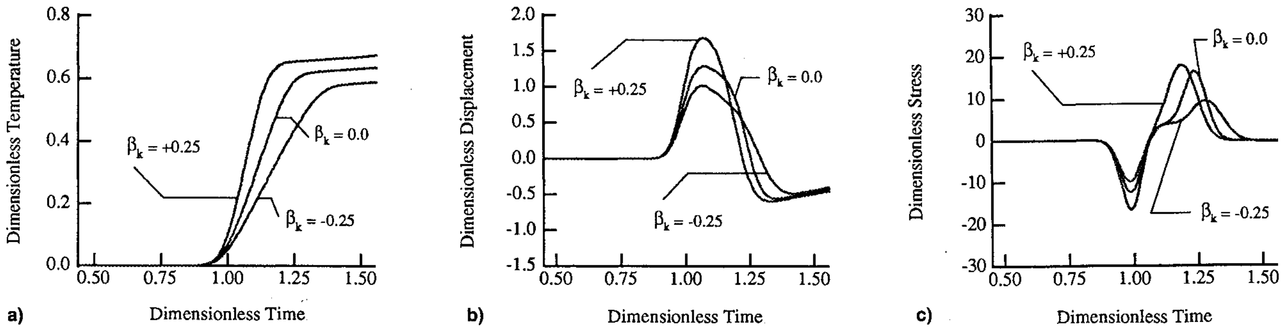

Fig. 4 Effect of temperature-dependent thermal conductivity on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a surface temperature linearly ramped to $\theta_s = 1$.

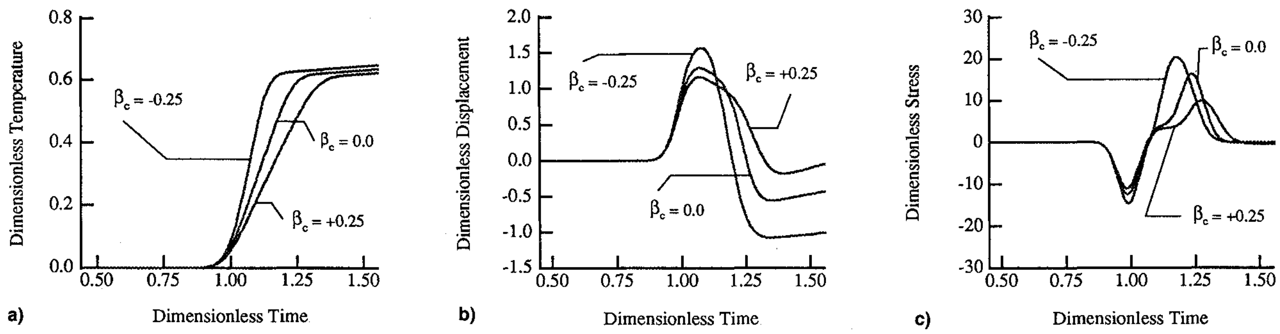

Fig. 5 Effect of temperature-dependent specific heat on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a surface temperature linearly ramped to $\theta_s = 1$.

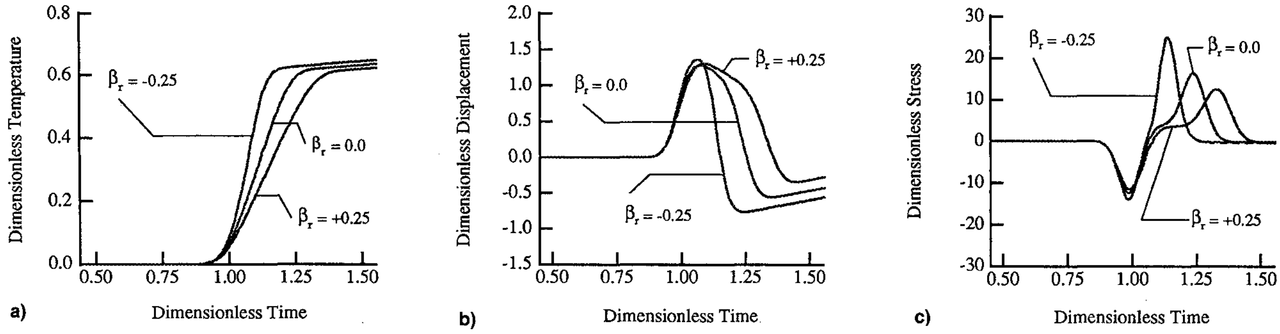

Fig. 6 Effect of temperature-dependent density on the a) temperature, b) displacement, and c) stress distribution in a slab at position $\eta = 1$ with a surface temperature linearly ramped to $\theta_s = 1$.

to the effect of the variable thermal conductivity which results in significantly different temperatures after the thermal wave has passed.

Ramp Increase in Surface Temperature

The effect of a linear ramp increase in the surface temperature on the temperature, displacement, and stress at $\eta = 1$ is also examined. The surface temperature at $\eta = 0$ linearly increases over time from $\xi = 0.0$ to $\xi = 0.25$, as opposed to the step increase in surface temperature studied previously. The effect of temperature-dependent thermal conductivity on the temperature, displacement, and stress is shown in Fig. 4. As a result of the temperature rise (Fig. 4a) being spread over time, and thus smaller thermal gradients, the maximum displacement and stress (Figs. 4b and 4c) are significantly lower than for the case of a step increase in surface temperature. In addition to a lower maximum value, the displacement and stress waves are also spread over a larger time interval. Unlike the step increase in surface temperature, there appears to be negligible phase shift in the maximum displacement with a ramped surface temperature. This, however, is not the case with the stress, where a phase shift is observed.

The effect of temperature-dependent specific heat on the temperature, displacement, and stress is shown in Fig. 5. As in Fig. 2, an increasing specific heat results in diffused temperature gradients (Fig. 5a), and thus lower magnitudes for the displacements and stresses (Figs. 5b and 5c) spread over a greater time interval when compared to the step increase case. Again, there appears to be a little phase shift in the maximum displacement due to the variable specific heat. However, the stress does experience a phase shift due to the variable specific heat.

The effect of temperature-dependent density on the temperature, displacement, and stress is shown in Fig. 6. Unlike the step increase case, variable density appears to have little effect on the maximum displacement (Fig. 6b) when the surface temperature increase is ramped. However, as in the step increase case, the variable density has a greater effect on the phase shift of the stress than does the variable specific heat. Though a ramped surface temperature effects the temperature, displacement, and stress differently than the step change in surface temperature, the qualitative effect of the temperature-dependent properties is similar.

Concluding Remarks

The effects of temperature-dependent thermal conductivity, specific heat, and density are studied for non-Fourier dynamic thermoelasticity problems. A generalization of MacCormack's explicit predictor-corrector method is used in the investigations. Both a step change and a linear ramp increase in the surface temperature are considered. It is observed that a thermal conductivity decreasing with temperature results in a reduced temperature gradient, and thus a reduced maximum displacement and stress. The opposite effect is observed to be true for both a decreasing specific heat and density. In addition to altering the magnitudes of the displacement and stress, the variable thermal properties also result in a phase shift of the displacement and stress waves. The variable density has a greater effect on the displacement and stress phase shift than does the variable specific heat. The ramp increase in surface temperature results in reduced temperature gradients. Consequently, the displacement and stress waves are of lower amplitude and spread over a greater time interval.

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